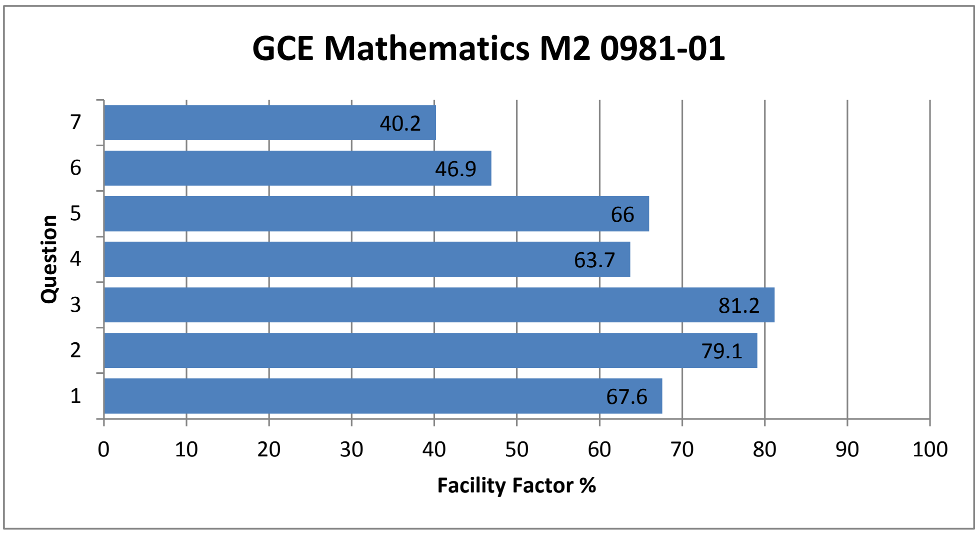
WJEC 2014 Online Exam Review

GCE Mathematics M2 0981-01

All Candidates' performance across questions

Question Title	*N*	*Mean*	*S D*	*Max Mark*	*F F*	*Attempt %*	
1	899	4.7	2.3	7	67.6	99.7	1
2	897	7.9	2.7	10	79.1	99.5	
3	899	8.1	2.9	10	81.2	99.7	3
4	886	6.4	3	10	63.7	98.2	4
5	896	8.6	4.3	13	66	99.3	
6	878	6.1	4	13	46.9	97.3	6
7	868	4.8	3.6	12	40.2	96.2	7



1. The diagram shows a piston, of mass 0·8 kg, enclosed in a horizontal tube and attached to a light spring of natural length 0·2 m and modulus of elasticity 625 N. The other end of the spring is fixed to the end of the tube at point *B*.

Initially, the piston is held at rest at a point *A* with the spring compressed a distance of 0·1 m, so that *AB* is the compressed length of the spring.

(a) Calculate the elastic energy stored in the spring. [2]

The piston is then released. During the subsequent motion, it is subjected to a resistance to motion of constant magnitude 46 N.

(b) Determine the velocity of the piston when the spring reaches its natural length. [5]

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Q1a) $\text{Elastic energy} = \frac{\lambda x^2}{2l} = \frac{625 \times 0.1^2}{2 \times 0.2} = 15.625\,J.$

Q1b)

Start

Elastic energy $= 15.625$.

End.

Kinetic $= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.8 \times v^2$.

$= 0.4v^2$.

start

potential $= mgh$.

$= 0.8 \times 9.8 \times 0.1$

$= 0.784$

End

$mgh = 0.8 \times 9.8 \times 0.2$

$= 1.568$.

$15.625 + 0.784 = 0.4v^2 + 1.568 - 46$

$16.034 = -44.432 + 0.4v^2$.

$60.466 = 0.4v^2$

$v^2 = 151.165$

$= 12.29$ (2dp).

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Q1a) Elastic energy $= \frac{\lambda x^2}{2l} = \frac{625 \times 0.1^2}{2 \times 0.2} = 15.625 J.$ 2

Q1b) Start

Elastic energy $= 15.625$.

End.

Kinetic $= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.8 \times V^2$

$= 0.4v^2$. B1

Start

potential $= mgh$.

$= 0.8 \times 9.8 \times 0.1$

$= 0.784$

End

$mgh = 0.8 \times 9.8 \times 0.2$

$= 1.568$. B1

B0

$15.625 + 0.784 = 0.4v^2 + 1.568 - ~~64~~ 46$

$16.034 = -44.432 + 0.4v^2$

$60.466 = 0.4v^2$ M0

$v^2 = 151.165$ A0

$= 12.29$ (2dp). A0

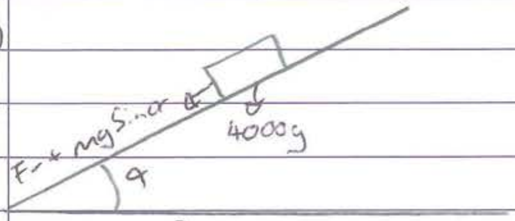
4

3. A vehicle of mass 4000 kg is travelling up a slope inclined at an angle α to the horizontal, where $\sin \alpha = \frac{2}{49}$. The engine of the vehicle is working at a constant rate of 90 kW.

(a) Calculate the resistance to the motion of the vehicle at the instant when its speed is $4{\cdot}8\,\text{ms}^{-1}$ and its acceleration is $1{\cdot}2\,\text{ms}^{-2}$. [6]

(b) Determine the maximum velocity of the vehicle when the resistance to motion has magnitude 12 800 N. [4]

3) a)



$\sin\alpha = \frac{2}{49}$

$P = Fv$

$90000 = F \times 4.8$

$\frac{90000}{4.8} = F$

$F = 18750$

$F - F_r = ma$

$F_r = F - ma$

$F_r = 18750 - 4000 \times 1.2$

$= \underline{13950\ N}$

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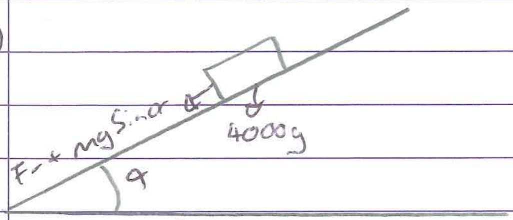
b)	At max velocity	
	$a = 0$	
	$\Rightarrow F = F_r$	
	$P = Fv$	
	$90000 = 12800\ v$	
	$\frac{90000}{12800} = v$	
	$v = 7.03125$	
	$= 7.03\ ms^{-1}$	

3) a)



$\sin \alpha = \frac{2}{49}$

$P = Fv$

$90000 = F \times 4.8$ ✓ M1

$\frac{90000}{4.8} = F$

$F = 18750$ ✓ A1

$F - F_r = ma$

$F_r = F - ma$ M0

$F_r = 18750 - 4000 \times 1.2$ component of wt. A0

$= \underline{13950\ N}$ A0

A0

Question number *Rhif y Cwestiwn*		Leave Blank *Gadewch yn wag*
b)	At max velocity	
	$a = 0$	M1
	$\Rightarrow F = F_r$	A0
	$P = Fv$	
	$90000 = 12800\,v$	
	$\frac{90000}{12800} = v$	B1
	$v = 7.03125$	A0
	$= 7.03\ ms^{-1}$	4

4. At time $t = 0$, an aeroplane A has position vector $(3\mathbf{i} + 5\mathbf{j} + 20\mathbf{k})$ m and is flying with constant velocity $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ ms^{-1}.
At time $t = 0$, another aeroplane B has position vector $(-2\mathbf{i} + x\mathbf{j} + 15\mathbf{k})$ m, and is flying with constant velocity $(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ ms^{-1}.

(a) Find expressions for the position vector of A and the position vector of B at time t s. [3]

(b) Determine an expression for AB^2, where AB is the distance between A and B at time t s. [4]

(c) Given that the shortest distance between A and B occurs at $t = 5$, calculate the value of x. [3]

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Q4)

a) $r_A = (3-t)\underline{i} + (5+2t)\underline{j} + (20+t)\underline{k}$

$r_b = (-2+3t)\underline{i} + (x-4t)\underline{j} + (15+2t)\underline{k}$

b) $AB = r_b - r_a$

$AB = (-5+4t)\underline{i} + ((x-5)-6t)\underline{j} + (-5+t)\underline{k}$

$AB^2 = (-5+4t)^2 + ((x-5)-6t)^2 + (-5+t)^2$

$AB^2 = 25 - 40t + (x-5)^2 - 12t(x-5) + 36t^2 + 25 - 10t + t^2$

$AB^2 = 50 - 50t + 37t^2 + x^2 - 10x + 25 - 12tx + 60t$

$AB^2 = 37t^2 + 75 + x^2 - 10x - 12tx + 10t$

c) $t = 5$

$\Rightarrow AB^2 = 37(5)^2 + 75 + x^2 - 10x - 75x + 50$

$AB^2 = 925 + 75 + x^2 - 85x + 50$

$AB^2 = x^2 - 85x + 1050$

shortest distance when $\frac{d}{dx}(AB^2) = 0$

$\frac{d}{dx}(AB^2) = 2x - 85$

$0 = 2x - 85$

$85 = 2x$

$\underline{x = 42.5}$

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Q4)

a) $r_A = (3-t)\underline{i} + (5+2t)\underline{j} + (20+t)\underline{k}$

$r_b = (-2+3t)\underline{i} + (x-4t)\underline{j} + (15+2t)\underline{k}$ 3

b) $AB = r_b - r_a$

$AB = (-5+4t)\underline{i} + ((x-5)-6t)\underline{j} + (-5+t)\underline{k}$

$AB^2 = (-5+4t)^2 + ((x-5)-6t)^2 + (-5+t)^2$ 4

$AB^2 = 25 - 40t + (x-5)^2 - 12t(x-5) + 36t^2 + 25 - 10t + t^2$

$AB^2 = 50 - \underset{50t}{} + 37t^2 + x^2 - 10x + 25 - 12tx + 60t$

$AB^2 = 37t^2 + 75 + x^2 - 10x - 12tx + 10t$

c) $t = 5$

$\Rightarrow AB^2 = 37(5)^2 + 75 + x^2 - 10x - 75x + 50$

$AB^2 = 925 + 75 + x^2 - 85x + 50$

$AB^2 = x^2 - 85x + 1050$

shortest distance
when $\frac{d}{dx}(AB^2) = 0$

$\frac{d}{dx}(AB^2) = 2x - 85$

$0 = 2x - 85$

$85 = 2x$

$x = 42.5$ 0

7

6. A particle of mass 3 kg moves on a horizontal plane. At time $t = 0$, the particle has position vector $-2\mathbf{i} + 3\mathbf{j}$ m, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along the x-axis and y-axis respectively. At time t s, the particle moves with velocity $\mathbf{v}$ ms^{-1} given by

$$\mathbf{v} = 4\sin 2t\,\mathbf{i} + 15\cos 5t\,\mathbf{j}.$$

(a) Find the magnitude of the force acting on the particle at time $t = \dfrac{3\pi}{2}$ s. [5]

(b) Determine the position vector of the particle at time t s. [4]

(c) Calculate the time and the distance of the particle from the origin when it crosses the y-axis for the first time. [4]

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6a) $a = \frac{d}{dt} v$

$= \frac{d}{dt} \; 4\sin 2t\,\underline{i} + 15\cos 5t\,\underline{j}$

$= 4\cos 2t \times 2\,\underline{i} - 15\sin 5t \times 5\,\underline{j}$

$= 8\cos 2t\,\underline{i} - 75\sin 5t$

$= \cancel{a}$ at $t = \frac{3\pi}{2}$

$a = 8\cos 2\left(\frac{3\pi}{2}\right) - 75\sin 5\left(\frac{3\pi}{2}\right)$

$= -8 - (-75)$

$= 67\,ms^{-2}$.

$F = ma$

$= 3 \times 67$

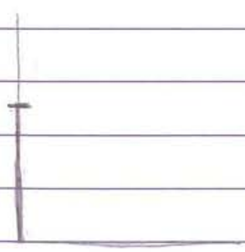
$= 201N$

b) $\underline{r} = (-2i + 3\underline{j}) + t(4\sin 2t\,\underline{i} + 15\cos 5t\,\underline{j})$

$= (-2\underline{i} + 3\underline{j}) + 4t\sin 2t\,\underline{i} + 15t\cos 5t\,\underline{j}$

$= (-2 + 4t\sin 2t)\underline{i} + (3 + 15t\cos 5t)\underline{j}$

c



x would be 0 $\therefore$ $\underline{i}$ would be 0

$r = 3t\underline{j}$

$r = 3 + 15\cos 5t$

$\cancel{\cos 5t = \frac{3}{15}}$

$3t\underline{j} = (3 + 15\cos 5t)\underline{j}$

$\cancel{15\cos 5t - 3t = -3}$

$\cancel{3(5\cos 5t - t) = -3}$

$\cancel{5\cos 5t - t = -1}$

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6a) $a = \frac{d}{dt} v$

$= \frac{d}{dt} \; 4\sin 2t \, \underline{i} + 15\cos 5t \, \underline{j}$

$= 4\cos 2t \times 2 \, \underline{i} - 15\sin 5t \times 5 \, \underline{j}$

$= 8\cos 2t \, \underline{i} - 75\sin 5t$ ✓ M1 A1

$=$ ~~a~~ at $t = \frac{3\pi}{2}$

$a = 8\cos 2\left(\frac{3\pi}{2}\right) i - 75 \sin 5\left(\frac{3\pi}{2}\right) j$ ✓✗ M0

$= -8 - (-75)$ M0

$= 67 \, ms^{-2}$ A0

$F = ma$

$= 3 \times 67$

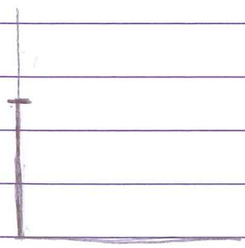
$= 201N$

b) $r = (-2i + 3j) + t\,(4\sin 2t\, \underline{i} + \cancel{15\cos 5t\, \underline{j}})$

$= (-2\underline{i} + 3\underline{j}) + 4t\sin 2t\, \underline{i} + 15t\cos 5t\, \underline{j}$

$= (-2 + 4t\sin 2t)\underline{i} + (3 + 15t\cos 5t)\underline{j}$ 0

c)



x would be 0 $\therefore$ $\underline{i}$ would be 0

$r = 3t\underline{j}$ not used.

$r = 3 + 15\cos 5t$

~~$\cos 5t = -\frac{3}{15}$~~

$3t\underline{j} = (3 + 15\cos 5t)\underline{j}$

~~$15\cos 5t - 3t = -3$~~

~~$3(5\cos 5t - t) = -3$~~

~~$5\cos 5t - t = -1$~~

7. One end of a light rod of length l metres is freely jointed to a fixed point *O* and the other end is attached to a particle of mass m kg. The particle is projected so that it describes a vertical circle. The speed of the particle at the highest point, u ms^{-1}, is a quarter of its speed at the lowest point of the circle.

(a) Show that $u^2 = \frac{4}{15}gl$. [3]

(b) When the rod is inclined at an angle θ to the **downward** vertical,

(i) find an expression for the tension in the rod in terms of m, g and θ.

(ii) determine the value of θ when the tension in the rod becomes zero. [9]

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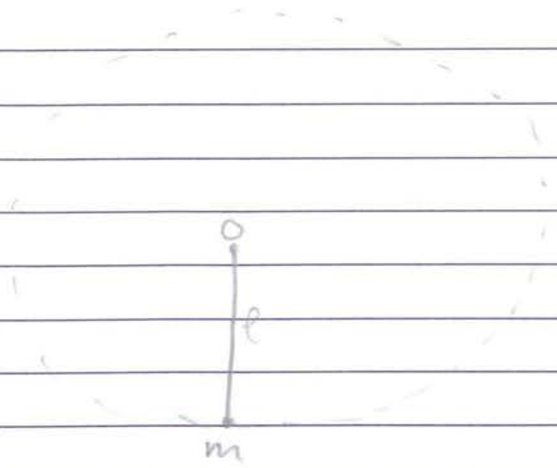


at highest point speed = $u\,ms^{-1}$
at lowest point speed = $4u\,ms^{-1}$

a) Initial energy $= \frac{1}{2}mv^2 + mg2l$

End energy $= \frac{1}{2}m(4v)^2$

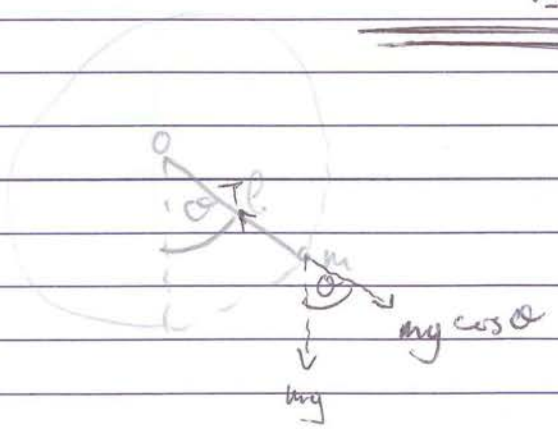
$$\frac{1}{2}m(4v)^2 = \frac{1}{2}mv^2 + mg\,2l$$

$$16v^2 = v^2 + 4gl$$

$$15v^2 = 4gl$$

$$v^2 = \frac{4}{15}gl$$

b)



i)

~~[illegible]~~ $T - mg\cos\theta = \dfrac{mv^2}{r}$

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$$T - mg\cos\theta = \frac{m\left(\frac{4}{15}gl\right)}{l}$$

$$T = \frac{4}{15}mg + mg\cos\theta$$

$$T = mg\left(\frac{4}{15} + \cos\theta\right)$$

(ii) $T = 0$.

$$mg\left(\frac{4}{15} + \cos\theta\right) = 0$$

$$\frac{4}{15} + \cos\theta = 0$$

$$\cos\theta = -\frac{4}{15}$$

$$\theta = 105.5^\circ$$

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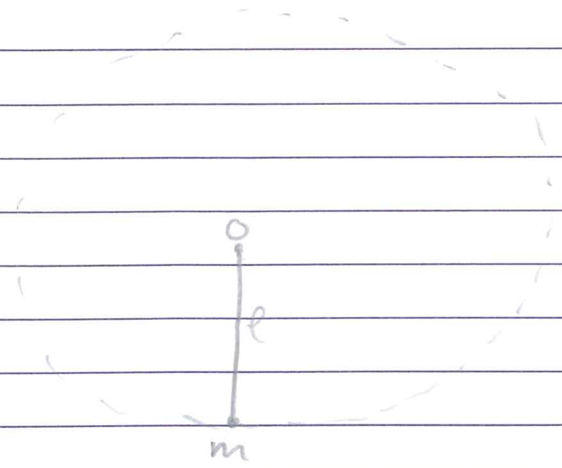


at highest point speed = $v \, ms^{-1}$
at lowest point speed = $4v \, ms^{-1}$

a) Initial energy $= \frac{1}{2} m v^2 + mg2l$

Final energy $= \frac{1}{2} m (4v)^2$

$$\frac{1}{2} m (4v)^2 = \frac{1}{2} m v^2 + mg2l$$

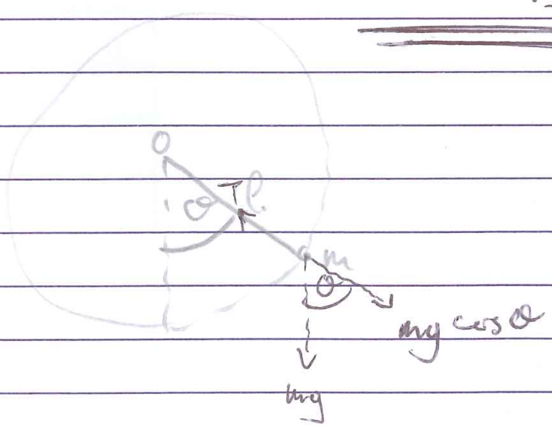
$$16v^2 = v^2 + 4gl$$

$$15v^2 = 4gl$$

$$v^2 = \frac{4}{15} gl$$

3

b)



i)

~~4T=2mg~~ $T - mg\cos\theta = \frac{mv^2}{r}$

M1

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$$T - mg\cos\theta = \frac{m\left(\frac{4}{15}gl\right)}{l}$$ A1 M0

$$T = \frac{4}{15}mg + mg\cos\theta$$ Energy M0 A0 A0

$$T = mg\left(\frac{4}{15} + \cos\theta\right)$$ A0

(ii) $T = 0$.

$$mg\left(\frac{4}{15} + \cos\theta\right) = 0$$

$$\frac{4}{15} + \cos\theta = 0$$

$$\cos\theta = -\frac{4}{15}$$ M1

$$\theta = 105.5^\circ$$ A1

7